Grade 4+: Computational Fluency

Collaborative Task:

Which One Doesn't Belong Routine for Formative Assessment http://wodb.ca/

If the *Which One Doesn't Belong* number routine is not yet familiar to your class please familiarize yourself with the web link included here. This is an instructional routine which has the potential for *'in the moment'* and *'after the moment'* (Peter Liljedhal). evidence of the multiplicative thinking process. The routine provides space to **listen** deeply to students' strategic thinking, **observe** their interactions and reasoning competencies and, if you wish, have a **product** as 'after the moment' evidence of reasoning and understanding of the computational fluency big idea for Grade 4.

Development of computational fluency and multiplicative thinking requires analysis of patterns and relations in multiplication and division.

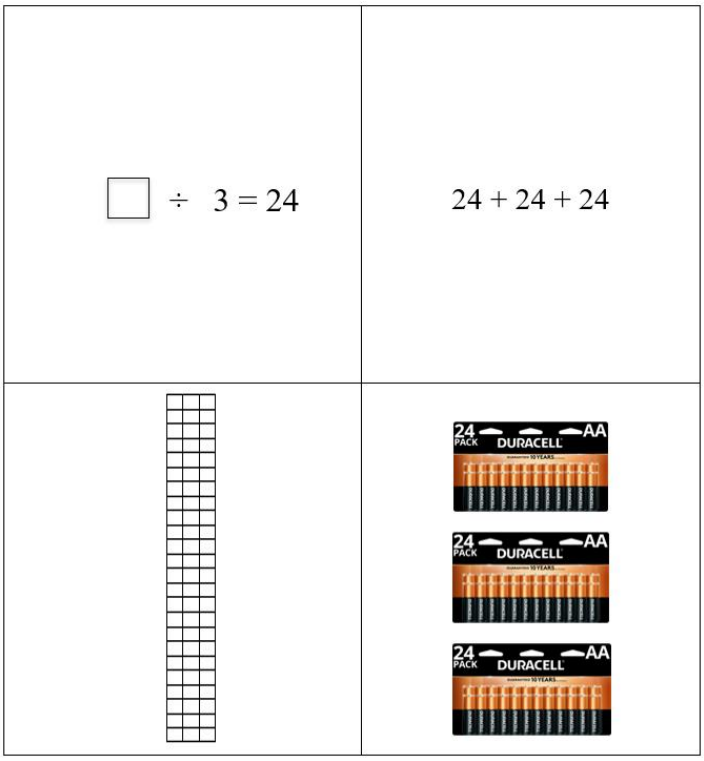
Project image or provide copies to groups.

Gathering 'in the moment' assessment evidence:

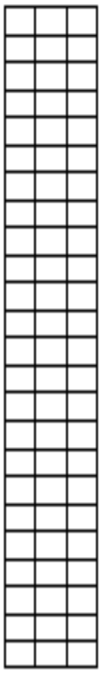
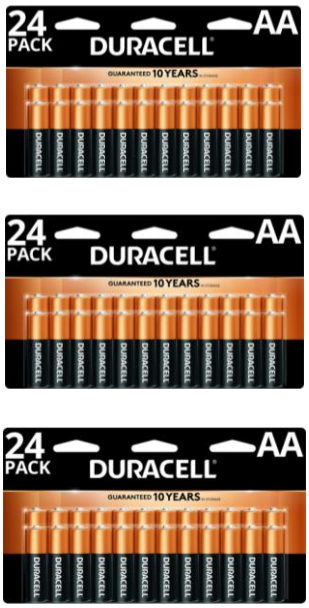
What mental/informal/intuitive strategies are students using?
How are they explaining their strategies?
Are they choosing efficient strategies?

Analyzing 'after the moment' evidence:

Is the formal written algorithm represented?
What do students know? What is difficult? What might be next steps?



Island Numeracy Assessment

$\square \div 3 = 24$	$24 + 24 + 24$
	

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Assessment Question	Answer Key
1. Find **at least three patterns** in the multiplication table. Describe the patterns. Consider how patterns show **multiplication or division.**	*The numbers in the '7 row' go up or down by 7 depending whether you are multiplying or dividing. If you add a number in the '2 row' to the number right below it, the total is in the 5 row since 2 of something and 3 more of that thing equals 5 of that thing. (additive reasoning)* *I notice in the '4 row' when I jump 4 squares of 4 that is 4 groups of 4 or 4X4 (multiplicative reasoning)* (from Marian Small Good Questions, WNCP edition Grades 4-6, page 27)
2. List three pairs of one-digit numbers that are easy to add in your head. List three pairs of one-digit numbers that are easy to multiply in your head. Explain your reasoning. Encourage students to demonstrate strategic reasoning with their choice of digits. *"What digits represent and extend my thinking?"*	9 + 1, 8 + 2, 7 + 7, 8+5 8+2 is easy since it makes 10 and I don't need to regroup. 8+5 - I take 2 from the 5 and make 10 so 10+3=13 2 X 2, 4 X 4, 4 X 5 4 X 5 is easy since I know how to skip count by 5s and I can just count 5,10,15,20 4X4 I know 2X4 is 8 so 4X4 is double 8

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

+	×

3. List **three examples** where dividing a two-digit number by a one-digit number is easy to do in your head. Tell why they would be easy for **each example.**	11 ÷ 1 = 11 12 ÷ 2 = 6 64 ÷ 8 = 8 44 ÷ 4 = 11 36 ÷ 6= 6 81 ÷ 9 = 9 12 ÷ 2 - we take an even number and it divides easily into groups of 2 . I know 2 groups of 6 makes 12. 81 ÷ 9 - I know 81 is a square number. An array dividing 81 into 9 rows and 9 columns. (from Marian Small Good Questions, WNCP edition Grades 4-6, page 27)
4. You divide one number by another in your head. The answer is almost 10. What might the numbers be?	45 divided by 4 72 divided by 8 108 divided by 12
5.	Demonstrating accuracy, fluency and flexibility (John Van de Walle) (from Marion Small, Open Questions 4-6, page 18)

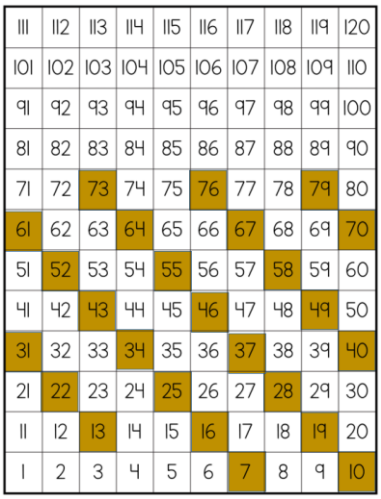
You add two numbers and the sum is close to 3 200 but not quite 3 200. What might the numbers be? 2 000 + 1 168 3000 + 199 168 + 3 000	You subtract two numbers and the difference is about 380. What might the numbers be? 700 - 350 400 – 19 500-119 999 - 628

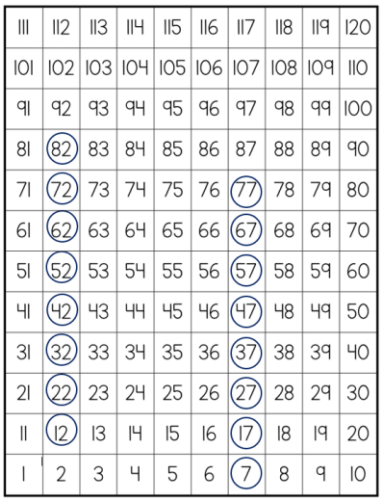
6. If you begin at 7 and skip count by 3's will you land on the number 79? Show and explain your thinking.

   If you begin at 7 and skip count by 5's, describe the pattern that emerges. Show and explain your thinking.

111	112	113	114	115	116	117	118	119	120
101	102	103	104	105	106	107	108	109	110
91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

111	112	113	114	115	116	117	118	119	120
101	102	103	104	105	106	107	108	109	110
91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10





7,10,13,16….79
79 is not divisible by 3 as the digital root is 7 not 3,6, or 9 but because the pattern starts on 7 you will land on 79. The pattern that emerges is diagonal.

7, 12,17,22…
The pattern will move up vertically in two columns on the 120s chart. 79 has a nine in the ones digit so in this pattern you will not land on it.

Up and down *pillars* or *columns* always end in 2 or 7. The pattern shows 5 more and 10 more.

7. You are arranging a class of students into equal-sized groups. Which class sizes have lots of possible arrangements? Which class sizes do not have as many possiblities? Explain your thinking.

Classes with prime numbers are not divisible in groups or arrangements.
24 is a class number with many potential arrangements. It has many factors; 2,3,4,6,8.
30 is easy to form equal groups; you can have groups of 2,3,5,6,10
27 is divisible by 3 and 9 only.

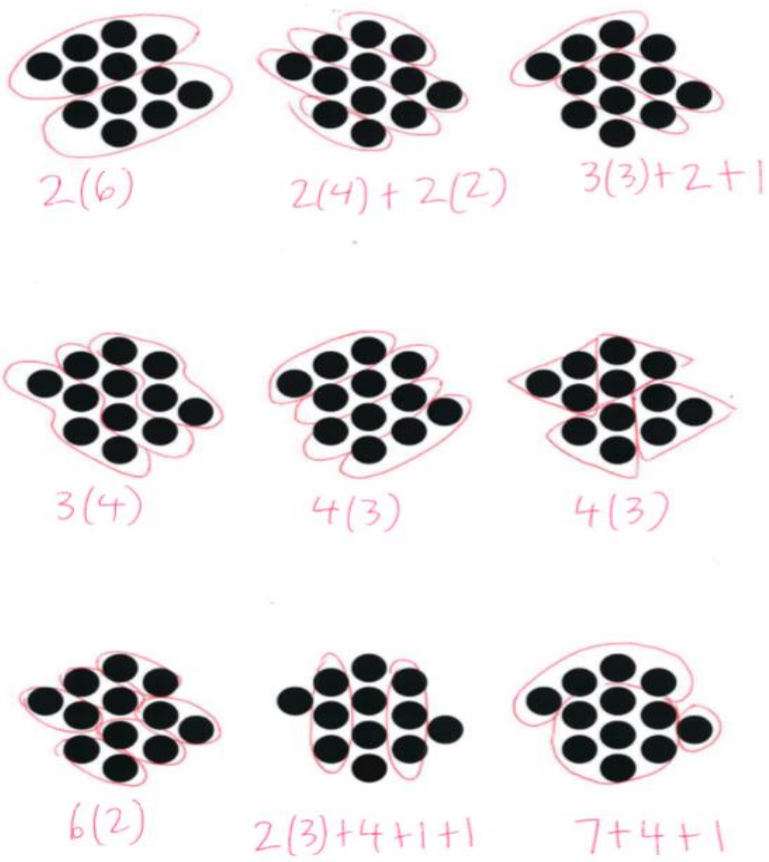
Marion Small, Open Questions grades 4-6

Grade 4+: Computational Fluency

Name: _____

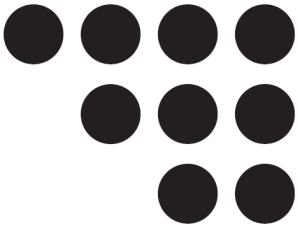
Performance Task

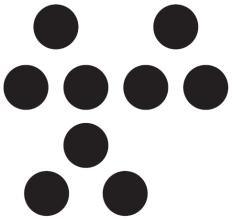
How many dots do you notice? How do you see them? Draw and share your thinking. Include a number sentence (equation) to match your thinking.

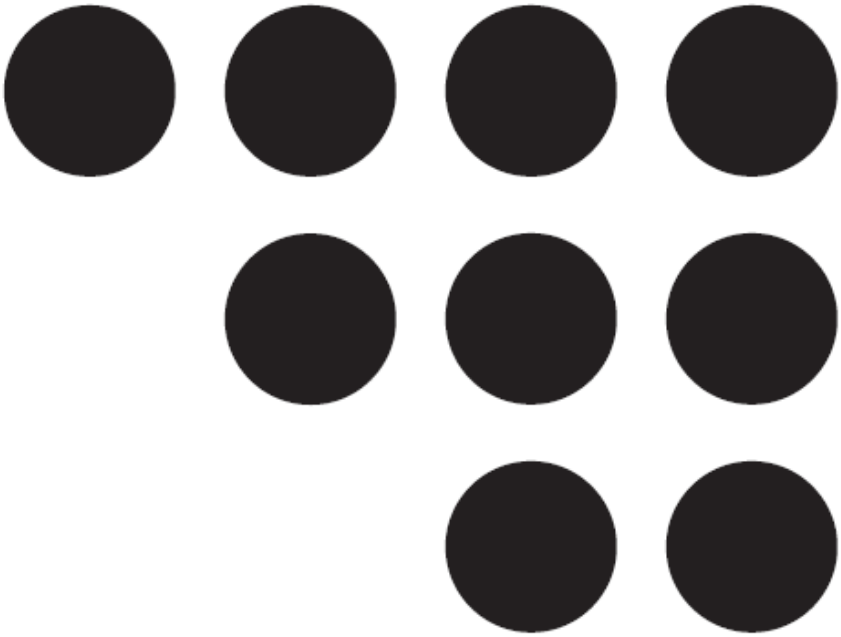


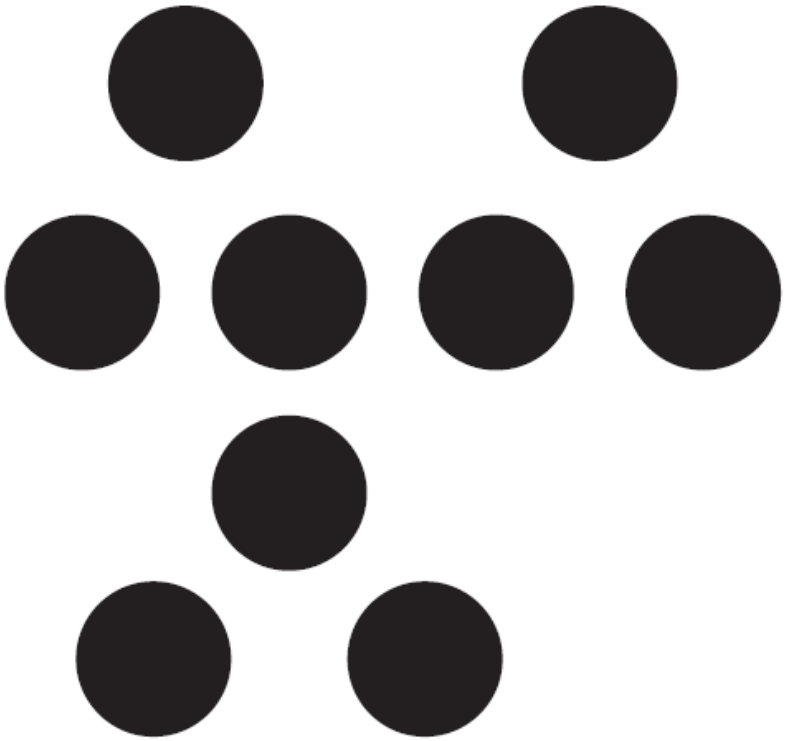
(adapted from Steve Wyborney's 'massive spaces to notice')

Model with a different dot arrangement with whole group first.









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Performance Task

Part I : There are two types of model race cars. The red cars take 3 AAA batteries and blue cars take 2 AA batteries. Imagine your team has one 48 pack of AAA and one 48 pack of AA batteries. You want to enter as many cars as possible in an upcoming race car tournament.

How many cars of each type could you enter in the race? 16 red cars that take 3 AAA batteries & 24 blue cars that take AA batteries for a total of 40 cars

Part II:

For the final race you are allowed to have 24 batteries as a team.
Red cars earn 3 points for a win.
Blue cars earn 5 points for a win.
What point totals are possible with just 24 batteries.

Teams earn 3 points for a AAA car win and 5 points for a AA car win. Your team earned 46 points.
What combinations using both AAA and AA entries are possible?

Your team is revved up and wants to go after top prize. Do you have enough points?
Possible combinations:
2AA and 12AAA 46 points
4AA is not possible as 16 is not a multiple of 3.
6AA is not possible as 16 is not a multiple of 3.
8AA and 2AAA 46 points

AA battery operated race cars' points earned will always be a multiple of 2 and points will always be multiples of 5.

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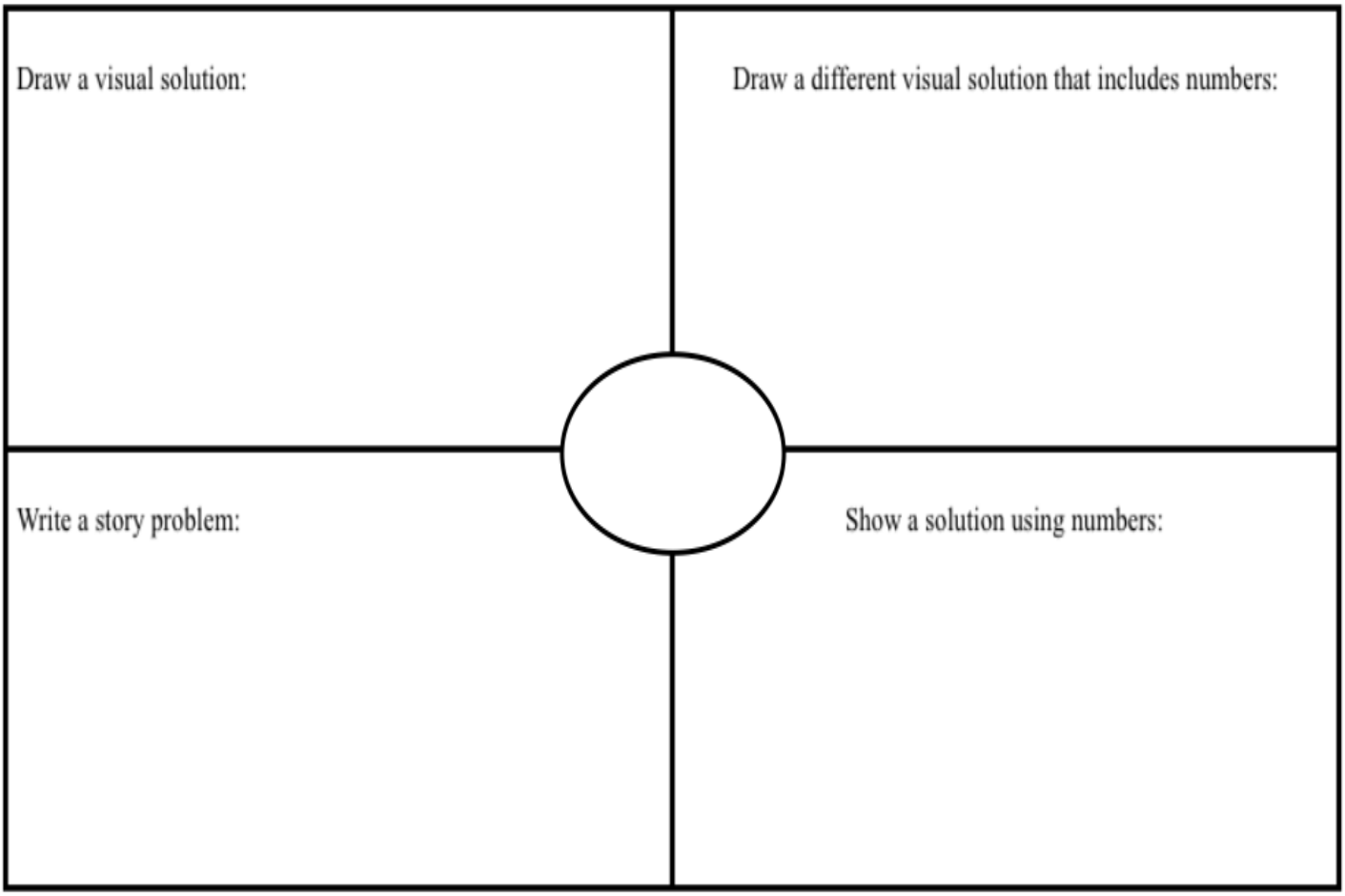
Performance Task

Choose one of the following equations to solve and represent in the following ways:

65 ÷ 4 **60 ÷ 15** **66 ÷ 5**



Consider using the *sugar packets 3 Act Task* as an introduction to this performance task.

A Dan Meyer 3 ACT lesson: http://threeacts.mrmeyer.com/sugarpackets/

Story problem 65 divided by 4 = 16 4g packets plus one more gram.

Bar Model representation: (decomposing number to visually show the remainder)

65				
16	16	16	16	1

Visual Solutions:

Arrays
Partitive/quotative models of groups of thinking. Knowing the # of sets (partitive), knowing the number (quantity) in each set (quotative)

Story Problems:

I have 65 stickers for an album. Each page of the album will have 16 stickers. With one sticker left over I begin a fifth page.

My class collects 65 bottles in our fundraiser. We divide the 65 bottles equally in 4 bags and there is one bottle left over.

There are 65 golfers for the golf tournament. They team up in groups of 4. There are 16 teams and one golfer looking for a pick-up game.

I have 65 dollars to spend on books. If each book costs 16 dollars I will have one dollar left over.

I have 65 books. I want to put the books in 4 boxes. Each box will have 16 books with one book remaining.

65 crayons could be put in 4 boxes with one crayon left over.

Numeric solutions:

$$10 + 10 + 10 + 10 = 40$$

$$6 + 6 + 6 + 6 = 24$$

$$(4X16) + 1 = 65$$

$$65 \div 4 = 16 \text{ remainder } 1 \qquad 4\sqrt{65}$$

$$\frac{65}{4} = 16 \text{ R } 1$$